A unified approach to percolation processes on multiplex networks

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Networks are a powerful tool to represent the heterogeneous structure of interactions in the study of complex systems. But in many cases there are multiple kinds of interactions, or multiple interacting sub-systems that cannot be adequately represented by a single network. Examples include financial infrastructure, informatic and ecological systems. There are many representations of multi-layer networks, appropriate in different Circumstances (see e. g. for a summary). We focus on multiplex networks, which are networks with a single set of nodes present in all layers, connected by a different type of edge (which may be represented by different colours) in each layer. Some interdependent networks, in which different layers have different sets of nodes as well, but the nodes arent connected between layers by interdependency links, are able to be captured by this construction. One of the fundamental structural properties of a network is its response to damage, that is, the percolation transition, where the giant connected component collapses. In multi-layer networks, interdependencies between layers can make a system more fragile. Damage to one element can trigger avalanches of failures that spread through the whole system. Typically a discontinuous hybrid phase transition is observed, similar to those observed in the network k-core or in bootstrap percolation in contrast to the continuous transition seen in classical percolation on a simplex network. Under a weaker definition of percolation, a more complex phase diagram emerges, with the possibility for both continuous and discontinuous transitions. When invulnerable or seed nodes are introduced, we can define activation and pruning processes, which have different phase diagrams. In a single-layer network (simplex), two nodes are connected if there is at least one path between them along the edges of the network. A group of connected nodes forms a cluster. The giant connected component (GCC) is a cluster which contains a finite fraction of the nodes in the network. The existence of such a giant component is synonymous with percolation. We can study its appearance by applying random damage to the network. A fraction p of nodes are removed, independently at random, and we check whether the remaining network contains a giant connected component. Typically the GCC appears linearly with a continuous second-order transition, although when the degree distribution is very broad (as in scale-free networks) the

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